Degenerate Neutrinos and CP Violation

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Abstract

We have studied mixing and masses of three left handed Majorana neutrinos in the model, which assumes exactly degenerate neutrino masses at some "neutrino unification" scale. Such a simple theoretical ansatz naturally leads to quasidegenerate neutrinos. The neutrino mass splittings induced by renormalization effects. In the model we found that the parameters of the neutrino physics (neutrino mass spectrum, mixing angles and CP violation phases) are strongly intercorrelated to each other. From these correlations we got strong bounds on the parameters which could be checked in the oscillation experiments.

In the present article we have studied the model of degenerate in mass neutrinos at some high energy in the general CP violation case.

We add to the basic Lagrangean the dimension–five operator

$$\frac{\lambda_0 \delta_{ab}}{M_X}(\phi \ell_a)(\ell_b \phi) \ + \ H.\,c. \tag{1}$$

where ℓ_a denote the three lepton doublets and ϕ is the standard Higgs doublet. The breaking of the electroweak symmetry due to a non-zero vacuum expectation value (VEV) $< \phi >$ will generate, in addition to the known SM masses, the seesaw-type neutrino mass operator M_ν

$$M_\nu = \frac{< \phi >^2}{M_X}\lambda_0 \ . \tag{2}$$

In the basis in which the charged lepton Yukawa couplings are diagonal (weak basis), $\lambda_0 \delta_{ij}$ gets transformed into

$$\Lambda = \lambda_0 U^* U^\dagger \tag{3}$$

where $U^\dagger$ is a matrix which is diagonalized the charged lepton Yukawa couplings.

In that basis we get

$$\mathcal{L} \supset \frac{\lambda_0 < \phi >^2}{M_X}\nu^T \nu - \frac{g}{\sqrt{2}}\overline{e_L}\gamma^\mu W_\mu^- U \nu + \ H.\,c. \tag{4}$$

In general, the mixing matrix K is characterized by three mixing angles and three CP violating phases, one Dirac plus two Majorana-type phases and can be written as

$$\begin{aligned} K \ &= \ \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\phi} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\phi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\phi} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\phi} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\phi} & c_{23}c_{13} \end{pmatrix} \\ &\cdot \ \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix} \end{aligned} \tag{5}$$

It is clear that with our ansatz (1) we have a freedom of performing any arbitrary real 3×3 rotation of the neutrino mass eigenstates [2]. However once the neutrino mass degeneracy is lifted by quantum corrections the mixing effects require the full matrix K with a non-trivial relationship amongst the mixing angles.

Now we turn to the renormalization effects. The renormalization group equations (RGEs) for the Λ coefficients characterizing the dimension-5 non-renormalizable terms can easily be written down both for the SM and MSSM. The flavour independent corrections to the $\Lambda = \emptyset(1)$ coefficients are irrelevant for our discussion. The flavour-dependent corrections are due to lepton Yukawa couplings and, to a good approximation, are determined by the τ Yukawa coupling. Moreover the supersymmetry can induce flavour-dependent threshold corrections associated with slepton mass splitting which can dominate over the τ Yukawa corrections.

The quantum corrections to our ansatz (1) in the weak basis are given by

$$M_\nu \;=\; c\; m_0 (1+R)\; U^* U^\dagger (1+R)) \tag{6}$$

where c is a common flavour-independent renormalization $\O(1)$ factor. The correction R (calculated in the electroweak charged lepton mass eigenstate basis) consists of two parts - the renormalization group correction and the electroweak scale threshold corrections. Assuming no lepton flavour violation in other sectors of the theory (e.g. in the case of supersymmetry) the matrix R is diagonal.

Without loss of generality R can be written as

$$R \;=\; r \begin{pmatrix} 1 & & \\ & 0 & \\ & & \epsilon \end{pmatrix} \tag{7}$$

In [1] it was shown that only when $|\epsilon| < 1$ the ansatz (1) leads to the correct phenomenological results.

The ordinary perturbation calculus tells that the off-diagonal entries of the matrix

$$A \;= K^T\; R\; K^* \;+\; K^\dagger\; R\; K \tag{8}$$

should be zero. We therefore require that

$$A_{12} = A_{13} = A_{23} = 0\ . \tag{9}$$

The corrected neutrino masses are given by

$$\begin{aligned} m_1 &= m\; (1 \;+\; rA_{11}) \\ m_2 &= m\; (1 \;+\; rA_{22}) \\ m_3 &= m\; (1 \;+\; rA_{33}) \end{aligned} \tag{10}$$

where $m = c\; m_0$.

Inserting K and R into eq. (8), taking into account eqs. (9) and (10) and assuming maximal atmospheric mixing angle we arrive to the following relations

$$\tan^2\theta_{12} \;=\; -\frac{\cos(\beta-\phi)\;\cos(\alpha-\beta)}{\cos\beta\;\cos(\alpha+\phi-\beta)}, \tag{11}$$

$$s_{13} = \left|\frac{\cos\alpha\;\cos(\alpha-\beta)}{\cos(\alpha+\phi-\beta)(\cos(\alpha-\phi)-\tan^2\theta_{12}\;\cos(\alpha+\phi))}\right|^{1/2}, \tag{12}$$

and

$$\frac{\Delta m^2_{21}}{\Delta m^2_{32}} \;=\; \frac{\cos(2\theta_{12})}{\sin^2\theta_{12}} + \frac{4\;\cos\alpha\;\cos\phi}{\cos(\alpha-\phi)-\tan^2\theta_{12}\;\cos(\alpha+\phi)}. \tag{13}$$

On the figs. we have pointplotted the predicted areas of the model on neutrino oscillation parameters.

References

[1] P. H. Chankowski, A. Ioannisian, S. Pokorski and J. W. F. Valle, Phys. Rev. Lett. **86** (2001) 3488 [arXiv:hep-ph/0011150].

[2] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Rev. Lett. **82**, 683 (1999) [arXiv:hep-ph/9810328].

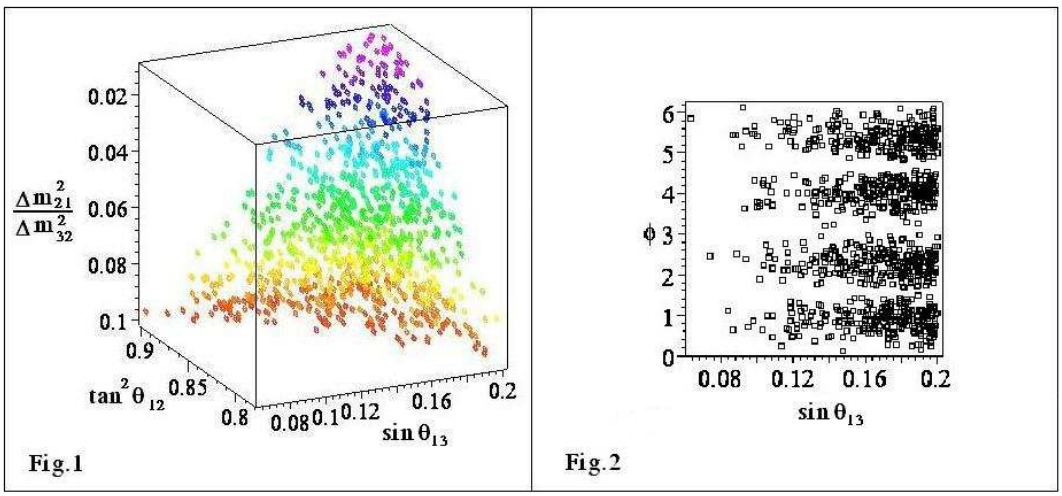


Fig. 1 - The area in the $\Delta m^2_{12}/\Delta m^2_{23}$ - $\tan^2\theta_{12}$ - $\sin\theta_{13}$ space predicted in our model. For a lower value of the solar mixing angle (θ_{12}) the model predicts a large ratio $\Delta m^2_{12}/\Delta m^2_{23}$ and a large "reactor" mixing angle (θ_{13}).

Fig. 2 - The areas in the ϕ - $\sin\theta_{13}$ plane predicted in our model.

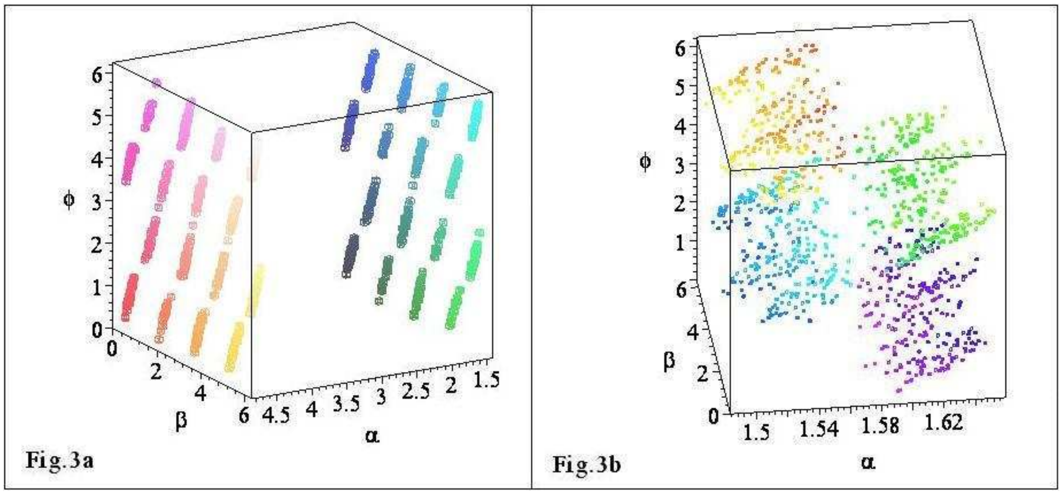
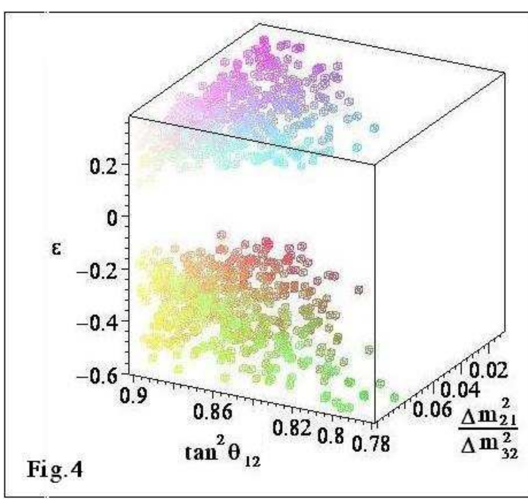


Fig. 3a[1] - The areas of the charged leptonic mixing matrix phases (Majorana α, β ; Dirac ϕ) predicted in our model. The solutions are only for $\alpha \simeq \pi/2+n\pi$ and $\phi-2\beta \simeq k\pi$ (k, n are integers).

Fig. 3b[1] - The areas of the charged leptonic mixing matrix phases (Majorana α, β ; Dirac ϕ) predicted in our model around value $\alpha \simeq \pi/2$.



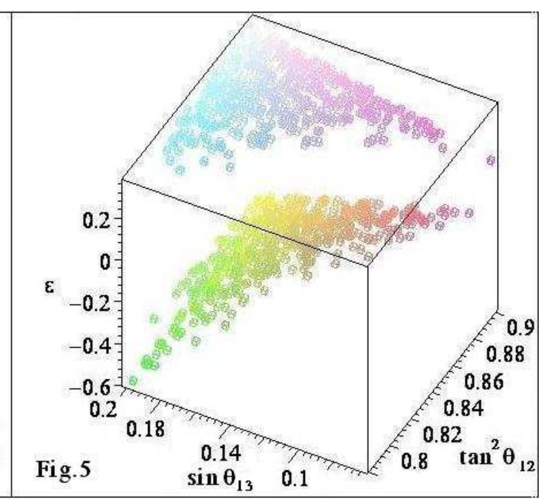


Fig. 4[1] - The parameter ϵ of our model depending on $\Delta m_{12}^2/\Delta m_{23}^2$ and $\tan^2\theta_{12}$.
Fig. 5[1] - The parameter ϵ of our model depending on $\sin\theta_{13}$ and $\tan^2\theta_{12}$.
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[1]The different colors are only used for better 3d visuality.